Level 4 Semester 2 Examination - February 2009

Answer 1

(a) The avalanche process is one that occurs in the breakdown of gaseous dielectrics and is based on the generation of successive ionising collisions leading to an avalanche.

Suppose a free electron exists (caused by some external effect such as radio-activity or cosmic radiation) in a gas where an electric field exists. If the field strength is sufficiently high, then it is likely to ionize a gas molecule by simple collision resulting in 2 free electrons and a positive ion. These 2 electrons will be able to cause further ionization by collision leading in general to 4 electrons and 3 positive ions. The process is cumulative, and the number of free electrons will go on increasing as they continue to move under the action of the electric field. The swarm of electrons and positive ions produced in this way is called an electron avalanche. In the space of a few millimetres, it may grow until it contains many millions of electrons, which is the avalanche.

(b) This type of breakdown mainly arises due to the added effect of the space-charge field of an avalanche and photo-electric ionization in the gas volume. While the Townsend mechanism predicts a very diffused form of discharge, in actual practice many discharges are found to be filamentary and irregular. The Streamer theory predicts the development of a spark discharge directly from a single avalanche. The space charge produced in the avalanche causes sufficient distortion of the electric field that those free electrons move towards the avalanche head, and in so doing generate further avalanches in a process that rapidly becomes cumulative.

As the electrons advance rapidly, the positive ions are left behind in a relatively slow-moving tail. The field will be enhanced in front of the head. Just behind the head the field between the electrons and the positive ions is in the opposite direction to the applied field and hence the resultant field strength is less. Again between the tail and the cathode the field is enhanced.

Due to the enhanced field between the head and the anode, the space charge increases, causing a further enhancement of the field around the anode. The process is very fast and the positive space charge extends to the cathode very rapidly resulting in the formation of a streamer.

On the application of an impulse voltage, a certain time elapses before actual breakdown occurs even though the applied voltage may be much more than sufficient to cause breakdown under static conditions.

In considering the time lag observed between the application of a voltage sufficient to cause breakdown and the actual breakdown the two basic processes of concern are (i) the appearance of avalanche initiating electrons and (ii) the temporal growth of current after the criterion for static breakdown is satisfied.

(c) SF₆ is an electronegative gas where there is a deficiency of electrons in the outer orbit or neutral molecules. Electro-negative gases are thus susceptible to electron attachment. Electron attachment to neutral molecules removes free electrons and thus gives gases very high dielectric strength. Such gases are suitable for indoor substations where space is a limitation.
(d) **Time lag characteristic**

![Diagram of time lag characteristic](image)

The time lag characteristic is the variation of the breakdown voltage with time of breakdown, and can be defined for a particular waveshape. The time lag characteristic based on the impulse waveform is shown.

For different amplitudes of the same waveform, the instant of breakdown and the peak amplitude of the prospective waveform is taken and joined to give the characteristic.

2 marks

(e) Impurities, which lead to the breakdown of commercial liquids below their intrinsic strength, can be divided into the following 3 categories.

(i) **Breakdown due to gaseous inclusions**

Gas or vapour bubbles may exist in impure liquid dielectrics, either formed from dissolved gasses, temperature and pressure variations, or other causes.

The electric field $E_b$ in a gas bubble which is immersed in a liquid of permittivity $\varepsilon_1$ is given by

$$E_b = \frac{3 \varepsilon_1}{2 \varepsilon_1 + 1} E_0$$

where $E_0$ is the field in the liquid in the absence of the bubble.

The electrostatic forces on the bubble cause it to get elongated in the direction of the electric field. The elongation continues, when sufficient electric field is applied, and at a critical length the gas inside the bubble (which has a lower breakdown strength) breaks down. This discharge causes decomposition of the liquid molecules and leads to total breakdown. 1 mark

(ii) **Breakdown due to liquid globules**

If an insulating liquid contains in suspension a globule of another liquid, then breakdown can result from instability of the globule in the electric field.

A spherical globule of liquid of permittivity $\varepsilon_2$ immersed in a liquid dielectric of permittivity $\varepsilon_1$, when it is subjected to an electric field between parallel electrodes, the field inside the globule would be given by

$$E = \frac{3 \varepsilon_1}{2 \varepsilon_1 + \varepsilon_2} E_0,$$

where $E_0$ is the field in the liquid in the absence of the globule.

The electrostatic forces cause the globule to elongate and take the shape of a prolate spheroid (i.e. an elongated spheroid). As the field is increased, the globule elongates so that the ratio $\gamma$ of the longer to the shorter diameter of the spheroid increases. For the same field $E$, the ratio $\gamma$ is a function of $\varepsilon_2/\varepsilon_1$. 2 marks
When \( \varepsilon_2 \gg \varepsilon_1 \) (generally when \( \varepsilon_2/\varepsilon_1 > 20 \)), and the field exceeds a critical value, no stable shape exists, and the globule keeps on elongating eventually causing bridging of the electrodes, and breakdown of the gap. When \( \varepsilon_2/\varepsilon_1 >> 20 \), the critical field at which the globule becomes unstable no longer depends on the ratio, and is given by \( E_{crit} \).

where

\[
E_{crit} = 1.542 \left( \frac{\sigma}{\varepsilon_1 R} \right)^{1/2} \text{ kV/cm}
\]

\( \sigma \) = surface tension of the globule (N/m)
\( \varepsilon_1 \) = relative permittivity of the insulating liquid
\( R \) = initial radius of globule (m).

Even a droplet of water even as small as 1 \( \mu \)m in radius (quite unobservable) can greatly reduce the breakdown strength of the liquid dielectric. Thus even submicroscopic sources of water, such as condensed breakdown products, or hygroscopic solid impurities, may greatly influence breakdown conditions. A globule which is unstable at an applied value of field elongates rapidly, and then electrode gap breakdown channels develop at the end of the globule. Propagation of the channels result in total breakdown.

\( \text{1 mark} \)

(iii) **Breakdown due to solid particles**

In commercial liquids, solid impurities cannot be avoided and will be present as fibres or as dispersed solid particles. If the impurity is considered to be a spherical particle of permittivity \( \varepsilon_2 \) and is present in a liquid dielectric of permittivity \( \varepsilon_1 \), it will experience a force

\[
F = \frac{1}{2} r^3 \varepsilon_0 \frac{\left( \varepsilon_2^2 - \varepsilon_1^2 \right)}{\varepsilon_2 + 2 \varepsilon_1} \Delta E^2
\]

where \( E = \) applied field, \( r = \) radius of particle.

Generally \( \varepsilon_2 > \varepsilon_1 \), so that the force would move the particle towards the regions of stronger field. Particles will continue to move in this way and will line up in the direction of the field. A stable chain of particles would be produced, which at a critical length may cause breakdown.

Because of the tendency to become contaminated, liquids are seldom used alone above 100 kV/cm in continuously energised equipment. However they may be used up to 1 MV/cm in conjunction with solids which can be made to act as barriers, preventing the line-up of solid impurities and localising bubbles which may form.

\( \text{1 mark} \)

(f) **Thermal Breakdown**

Heat is generated continuously in electrically stressed insulation by dielectric losses, which is transferred to the surrounding medium by conduction through the solid dielectric and by radiation from its outer surfaces. If the heat generated exceeds the heat lost to the surroundings, the temperature of the insulation increases.
The simplest case is where the loss of heat by cooling is linearly related to the temperature rise above surroundings, and the heat generated is independent of temperature. (i.e. the resistivity and the loss angle do not vary with temperature).

Heat lost = $k(\theta - \theta_0)$, where $\theta = \text{ambient temperature}$

Equilibrium will be reached at a temperature $\theta_1$ where the heat generated is equal to the heat lost to the surroundings, as shown.

In practice, although the heat lost may be considered somewhat linear, the heat generated increases rapidly with temperature, and at certain values of electric field no stable state exists where the heat lost is equal to the heat generated so that the material breaks down thermally. The rapid increase is due to the fact that with rise in temperature, the loss angle of the dielectric increases in accordance with an exponential law ($\text{loss} \propto e^{-A/T}$, where $T$ is the absolute temperature).

Figure shows the variation of heat generated by a device for 2 different applied fields and the heat lost from the device with temperature.

For the field $E_2$, a stable temperature $\theta_A$ exists (provided the temperature is not allowed to reach $\theta_B$).

For the field $E_1$, the heat generated is always greater than the heat lost so that the temperature would keep increasing until breakdown occurs.

The maximum voltage a given insulating material can withstand cannot be increased indefinitely simply by increasing its thickness. Owing to thermal effects, there is an upper limit of voltage $V_b$, beyond which it is not possible to go without thermal instability. This is because with thick insulation, the internal temperature is little affected by the surface conditions. Usually, in the practical use of insulating materials, $V_b$ is a limiting factor only for high-temperature operation, or at high frequency failures.
Answer 2

(a) 132 kV, 3, 50Hz transmission line \( \rightarrow V_{\text{phase}} = 132/\sqrt{3} = 76.21 \text{ kV} \)
diameter = 18 mm, equally spaced ACSR conductors \( \rightarrow r = 0.9 \text{ cm} \)
operating temperature = 40°C
corona inception at 5% higher = 1.05\times76.21 = 80.02 \text{ kV}
Assuming the relationship between the electric field \( \xi \) and the line-to-neutral voltage \( E_0 \) as
\[
\xi = \frac{E_0}{r \log_e \frac{d}{r}}
\]
Assuming breakdown strength of air at normal temperature and pressure conditions as 30 kV/cm
This corresponds to peak value, giving a corresponding rms value as 30/\sqrt{2} = 21.21 \text{ kV}
Thus \( \xi_{rms} = 21.2 = \frac{E_{0,rms}}{r \log_e \frac{d}{r}} \), where \( E_{0,rms} \) is in kV and \( r \) is in cm
Corona is affected by the air density
The air density correction factor \( \delta \), with \( p \) the pressure in torr and \( t \) the temperature in °C is
\[
\delta = \frac{p}{760} \cdot \frac{273 + 20}{273 + t} = \frac{760}{760} \cdot \frac{273 + 20}{273 + 40} = 0.936
\]
When the surface of the conductor is irregular, it is more liable to corona. Thus an irregularity factor \( m_0 \) is introduced to account for this reduction.
Typical values of this factor range for a multi-strand conductor is \( m_0 = 0.90 \)
The disruptive critical voltage can then be written as
\[
E_{0,rms} = 21.2 \delta m_0 r \log_e \left( \frac{d}{r} \right) \text{ kV to neutral}
\]
i.e. 80.02 = 21.2\times0.936\times0.90\times0.9\times \log_e \left( \frac{d}{0.9} \right)
\[\log_e \left( \frac{d}{0.9} \right) = 4.9785 \rightarrow \frac{d}{0.9} = 145.3 \]
d = 0.9\times145.3 = 130.7 \text{ cm}
Required spacing between conductors for an equilateral arrangement = 1.3 m

(b) Cascade arrangement of transformers

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Figure shows a typical cascade arrangement of transformers used to obtain up to 300 kV from three units each rated at 100 kV insulation. The low voltage winding is connected to the primary of the first transformer, and this is connected to the transformer tank which is earthed. One end of the high voltage winding is also earthed through the tank. The high voltage end and a tapping near this end is taken out at the top of the transformer through a bushing, and forms the primary of the second transformer.

One end of this winding is connected to the tank of the second transformer to maintain the tank at high voltage. The secondary of this transformer too has one end connected to the tank and at the other end the next cascaded transformer is fed.

This cascade arrangement can be continued further if a still higher voltage is required. In the cascade arrangement shown, each transformer needs only to be insulated for 100 kV, and hence the transformer can be relatively small. If a 300 kV transformer had to be used instead, the size would be massive. High voltage transformers for testing purposes are designed purposely to have a poor regulation. This is to ensure that when the secondary of the transformer is short circuited (as will commonly happen in flash-over tests of insulation), the current would not increase to too high a value and to reduce the cost. In practice, an additional series resistance (commonly a water resistance) is also used in such cases to limit the current and prevent possible damage to the transformer.

(c) 100 kVA, 230V/50 kV, 50 Hz testing transformer, 10% leakage reactance, 2% winding resistance, Q factor = 20, cable capacitance = 100 nF

Base Z = \( \frac{V^2}{V \cdot A} = \frac{(50 \times 10^3)^2}{100 \times 10^1} = 25,000 \Omega \)

\( x_t = 10\% = 25,000 \times 0.10 = 2,500 \Omega, \)

\( L_t = \frac{2500}{100 \pi} = 7.958 \text{ H}, \)

\( r_t = 2\% = 25,000 \times 0.02 = 500 \Omega \)

For resonance, \( \omega_c = \frac{1}{\sqrt{L_c C}}, \)

\( \frac{1}{(2 \pi \times 50)^2 \times 100 \times 10^{-9}} = 101.3 \text{ H} \rightarrow L \)

\( L = 101.3 - 7.96 = 93.3 \text{ H} \)

\( Q = 20 = \frac{L \omega_c}{r} \rightarrow r = \frac{93.3 \times 100 \pi}{20} = 1,465 \Omega, \)

\( I = V C \omega = 300 \times 10^3 \times 100 \times 10^{-9} \times 100 \pi = 9.425 \text{ A} \)

\( V_{in} (hv) = 9.425 \times (1465 + 500) = 18,520 \text{ V} \rightarrow V_{in} (lv) = 18,520 \times \frac{230}{50000} = 85.2 \text{ V} \)

(d) Capacitor charging method
In the positive half cycle, the capacitor charges up to the peak value, and when the voltage falls it discharges (very slightly) through the milliammeter, so that the voltage across the capacitor is very nearly a constant at the peak value and the current is thus proportional to the peak value.

(The time constant RC of the circuit must be very high in comparison to the period of the applied voltage).

3 marks

Answer 3

(a)

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Breakdown stress $\xi_{\text{max}}$ (kV/cm)</td>
<td>200</td>
<td>180</td>
<td>250</td>
</tr>
<tr>
<td>Relative Permittivity $\varepsilon_r$</td>
<td>4.4</td>
<td>3.2</td>
<td>2.8</td>
</tr>
<tr>
<td>$\frac{\xi_{\text{max}} \varepsilon_r}{2}$</td>
<td>880/2$^*$</td>
<td>576/2$^*$</td>
<td>700/2$^*$</td>
</tr>
</tbody>
</table>

(using safety factor of 2)

$V = 66$ kV, $V$ phase to neutral $= 72.5/\sqrt{3} = 41.858$ kV, $r = 10$mm

Maximum field occurs at the minimum radius within each section

Thus the maximum electric fields in each section are given by

$$\xi_{\text{max}1} = \frac{q}{2 \pi \varepsilon_0 \varepsilon_1 r}, \xi_{\text{max}2} = \frac{q}{2 \pi \varepsilon_0 \varepsilon_2 r_1}, \xi_{\text{max}3} = \frac{q}{2 \pi \varepsilon_0 \varepsilon_3 r_2}$$

i.e. $\xi_{\text{m1}} \varepsilon_1 r_1 = \xi_{\text{m2}} \varepsilon_2 r_1 = \xi_{\text{m3}} \varepsilon_3 r_3$

$\therefore \xi_{\text{m1}} \varepsilon_1 > \xi_{\text{m2}} \varepsilon_2 > \xi_{\text{m3}} \varepsilon_3$

$\therefore 440 \times 10 = 350 \times r_1 = 288 \times r_2$

$\therefore r_1 = 12.57 \text{ mm}, r_2 = 15.28 \text{ mm}$

maximum operating voltage (peak) $= \Sigma \xi_m r \ln R/r = 41.858 \times \sqrt{2} = 59.20$ kV

Thus

$$59.20 = \sum \xi_{\text{max}} r_{\text{min}} \ln \left(\frac{r_{\text{max}}}{r_{\text{min}}}\right)$$

i.e. $59.20 = \frac{200}{2} \times 1.0 \times \ln \left(\frac{1.257}{1.0}\right) + \frac{250}{2} \times 1.257 \times \ln \left(\frac{1.528}{1.257}\right) + \frac{180}{2} \times 1.528 \times \ln \left(\frac{R}{1.528}\right)$

$$59.20 = 22.87 + 24.40 + 137.52 \times \ln(R/1.528)$$

$\ln(R/1.528) = 11.93/137.52 = 0.0868$

$R/1.528 = 1.091 \rightarrow R = 1.666$ mm

$\therefore$ thickness are

$t_1 = 12.57 - 10 = 2.57$ mm,
$t_2 = 15.28 - 12.57 = 2.71$ mm
$t_3 = 16.66 - 15.28 = 1.38$ mm

7 marks
(b) The electric flux lines and the equipotential lines are perpendicular to each other. Further for constant differences, $\Delta \varphi$ (or $\Delta q$) and $\Delta V$ are constants. Thus for the elemental figure shown, capacitance is constant.

$$C = \frac{A \varepsilon}{d} = \frac{\Delta y \cdot I \cdot \varepsilon}{\Delta x} = \text{constant}$$

i.e. $\Delta y / \Delta x = \text{constant}$ (usually chosen as 1 for convenience of drawing). Thus curvilinear squares are formed in the sketch.

Considering the instant of time when A is at peak (1.0) and B and C are at half value (-0.5) the flux lines and the equipotential lines may be drawn as follows.

(c) Cross bonding of cables

When three single phase cables are used in power transmission, currents are induced in the sheaths and lead to sheath circulating currents and power loss. These may be substantially...
reduced, and the current rating of the cable increased by cross bonding of the cables. Cross bonding of cables are done except for very short lengths of cable.

The continuity of each cable sheath is broken at regular intervals; the cables between two adjacent discontinuities being a minor section. 3 minor sections make up a major section, where the sheaths are solidly bonded together and to earth. A residual sheath voltage exists, and the desired balance, giving negligible sheath voltage between the solid grounded positions is achieved by transposing the cables at each cross-bonded section. To prevent excessive voltage build up at the cross bonded points, especially during faults, these points are earthed through non-linear resistors which limit voltage build up. The cable is also transposed. 3 marks

Answer 4

(a) When two parallel conducting plates (cross section area A and spacing x) are charged q and have a potential difference V, then the energy stored in the pair of plates is given by

\[
\text{Energy stored } W = \frac{1}{2} C V^2
\]

so that \( \frac{dW}{dx} = \frac{1}{2} V^2 \frac{dC}{dx} = F \)

\[\therefore \text{ Force } F = \frac{1}{2} V^2 \frac{dC}{dx} \text{ N} \]

Thus the deflecting torque of an electrostatic voltmeter is proportional to the product of the square of the applied voltage and the rate of change of capacitance. 3 marks

for uniform field Capacitance \( C = \frac{A \varepsilon}{x} \) so that \( \frac{dC}{dx} = \frac{A \varepsilon}{x^2} \)

\[\therefore F = -\frac{1}{2} A \varepsilon \frac{V^2}{x^2} \text{ N} \]
The force of attraction is proportional to the square of the potential difference applied, provided the variation in \( x \) is small, so that the meter reads the square value (or can be marked to read the rms value). In this meter, the electrostatic force is balanced by a spring force as shown.

The right hand electrode forms the high voltage plate, while the centre portion of the left hand disc is cut away and encloses a small disc which is movable and is geared to the pointer of the instrument. The purpose of this is to keep the field roughly constant as movement occurs only on a small section.

The range of the instrument can be altered by setting the right hand disc at pre-marked distances. The two large discs form adequate protection for the working parts of the instrument against external electrostatic disturbances. These electrostatic instruments can be used to measure both a.c. and d.c. voltages.

(b) A cable used to connect the test waveform to the oscilloscope will necessarily have reflections occurring both at the potential divider end and the oscilloscope end. Thus matching needs to be done.

In this case, the cable is matched at both ends, so that there is no reflection at either end.

This arrangement reduces to a minimum the irregularities produced by the cable circuit.

Matching the cable at the receiving end would require the impedance seen by the surge at \( E_4 \) to be equal to the cable surge impedance \( Z_0 \). Also the sending end would require the effective impedance seen at \( E_2 \) from the cable side to match the cable.

Thus for perfect matching at receiving end, \( R_4 = Z_0 \)

and for perfect matching at sending end, \( R_3 + R_1 / / R_2 = Z_0 \)

since \( R_1 \gg R_2 \), this gives \( R_3 + R_2 = Z_0 \)

Thus

at \( E_2 \), the equivalent impedance \( Z_1 \) to earth is given by

\[
Z_1 = R_2 / / (R_3 + Z_0) = \frac{R_2(R_3 + Z_0)}{R_2 + R_3 + Z_0} = \frac{R_2(R_3 + Z_0)}{2Z_0}, \quad \therefore Z_0 = R_2 + R_3
\]

\[
\text{voltage at junction } E_2 = \frac{Z_1}{Z_1 + R_1} \cdot E_1 = \frac{R_2(R_3 + Z_0)}{2Z_0(Z_1 + R_1)} \cdot E_1
\]

so that

\[
E_3 = \frac{Z_0}{R_3 + Z_0} \cdot E_2 = \frac{Z_0}{(R_3 + Z_0)} \cdot \frac{R_2(R_3 + Z_0)}{2Z_0(Z_1 + R_1)} \cdot E_1 = \frac{R_2}{2(Z_1 + R_1)} \cdot E_1
\]

Due to perfect matching at the receiving end, this is transmitted without any reflections.

\[
\therefore E_4 = E_3 = \frac{R_2}{2(R_1 + Z_1)} \cdot E_1
\]

Thus an undistorted waveform is obtained with ratio \( \frac{R_2}{2(R_1 + Z_1)} \).  

Prepared by JRL – University of Moratuwa – February 2009
(c) Dielectric loss measurement using x-y mode in Oscilloscope

In an oscilloscope, if two alternating voltages of the same frequency are applied to the x-plate and y-plate, the resulting figure will be an ellipse. When the two voltages are in phase, the figure will be a straight line with an enclosed area of zero. As the phase angle difference increases, the area increases and reaches a maximum when the phase angle difference is 90°.

This property is made use of in dielectric loss measurements. A potential difference proportional to the applied voltage is applied to one pair of plates and a potential difference proportional to the integral of the current through the dielectric is applied to the other pair. Since the loss is to be measured in a dielectric sample, a lossless large capacitor is connected in series with the sample.

The voltages across the capacitor and across the sample are applied across the two plates. The area of the ellipse thus formed is proportional to the power loss in the dielectric. If the power loss in the dielectric is zero, the figure traced out on the oscilloscope would be a straight line.

The use of the standard capacitor C ensures that the voltage across it is 90° out of phase with the current. Hence the angle on which the area of the ellipse depends is not the power factor angle but the loss angle.

\[
\text{Power loss in } C_s = V_2 I_s \sin \delta
\]

The y-deflection on the oscilloscope is proportion to \( V_1 = V_{1m} \sin (\omega t - \delta) \) and the x-deflection is proportional to \( V_2 = V_{2m} \sin \omega t \) which is taken as the reference.

i.e. \( y = a \cdot V_{1m} \sin (\omega t - \delta) \)
\[
= a \cdot \left( \frac{I_{sm}}{\omega C} \right) \sin (\omega t - \delta)
\]
and \( x = b \cdot V_{2m} \sin \omega t \)

where \( a, b \) are constants.

The area of the ellipse traced out on the oscilloscope screen is given by

\[
A = \int y \cdot dx = \int_0^{\frac{T}{2}} a \cdot \frac{I_{sm}}{\omega C} \cdot \sin(\omega t - \delta) \cdot b \cdot V_{2m} \cdot \omega \cdot \cos \omega t \cdot dt
\]
\[
= \frac{a \cdot b \cdot 2 \pi}{\omega} \cdot \frac{I_s V_2}{C} \sin \delta
\]

It is thus seen that the area of the ellipse is proportional to the power loss.

Answer 5

(a) Since the losses in the high voltage standard capacitor and in the high voltage test capacitor are extremely low, the capacitive balance condition may be obtained without considering losses.
Thus
\[
\frac{C_2}{C_i} = \frac{Q}{S} \quad \text{giving} \quad C_1 = \frac{S}{Q} C_2 = \frac{100}{100} \times 20 = 20 \text{ pF}
\]

\[
C_1 = \frac{A \varepsilon_r \varepsilon_r}{d}
\]

\[
\varepsilon_r = \frac{C \cdot d}{\varepsilon_0 \cdot A} = \frac{20 \times 10^{-12} \times 0.02}{8.854 \times 10^{-12} \times 0.01} = 4.51
\]

for balance of an a.c. bridge, the phase angles must also balance.

impedance angle = \(-\varphi\),

loss angle = \(\pi/2 - \varphi = \delta\)

Thus
\[
\frac{Z_1 \angle (-\pi/2 + \delta_1)}{Z_2 \angle (-\pi/2 + \delta_2)} = \frac{Z_4 \angle 0}{Z_3 \angle -\theta_3} \quad \text{giving} \quad (-\pi/2 + \delta_1) - (-\pi/2 + \delta_2) = 0 - (-\theta_3)
\]

i.e. \(\delta_1 - \delta_2 = \theta_3\), \(\delta_1 = \delta_2 + \theta_3\)

\[
\tan \delta_1 \approx \tan \delta_2 + \tan \theta = 0.001 + 100\pi \times 200 \times 10^{-9} \times 100 = 0.00728
\]

\[
\frac{1}{\omega C_1 P} = 0.00728 \rightarrow P = 1/(100\pi \times 20 \times 10^{-12} \times 0.00728) = 21,862 \text{ M}\Omega
\]

\[
\rho = \frac{R \cdot A}{l} = \frac{21.862 \times 10^9 \times 0.01}{0.02} = 10.9 \times 10^9 \text{ \Omega m}
\]

(b)
(c) Test Cell used in the Measurement of dielectric constant and loss tangent of an insulating liquid

The test cell used in the measurement consists of a brass cell inside which is suspended a brass electrode from a perspex cover. The outer cell is the earthed electrode, and there is a gap of 3 mm all round between this and the inner brass electrode. Since the electrodes are near each other, the stray capacitance must be considered.

The test cell is connected in parallel with a variable capacitor and made part of a constant current resonant circuit. The inner brass electrode of the test cell can be removed to obtain and eliminate the stray capacitance.

The test cell permits replacement of the 3 mm air gap by the insulating liquid permitting comparison of capacitances to determine the dielectric constant through the measurement of capacitance change at resonance.

(d) The circuit is then de-tuned to the half-power points (voltage corresponding to $1/\sqrt{2}$) from which the value of the effective Q-factor is determined.

If $C_+$ and $C_-$ are the values at the half power points, then it can be shown that the Q factor at resonance can be obtained from

$$Q = \frac{C_+ + C_-}{C_+ - C_-} = \frac{2C_+ (\Delta C_+ + \Delta C_-)}{\Delta C_+ - \Delta C_-},$$

where $\Delta C_+$ and $\Delta C_-$ are the variations at the half-power points.

Usually $Q$ is high, and $\Delta C_+ = \Delta C_- = \Delta C$, so that $Q = \frac{C}{\Delta C}$.

(e) The tests on insulators can be divided into three groups. These are the type tests, sample tests and the routine tests.

**Type tests** are done to determine whether the particular design is suitable for the purpose. A sample of a type test for an insulator is described below.

**One-minute Rain test**: The insulator is sprayed throughout the test with artificial rain drawn from a source of supply at a temperature within 10°C of the ambient temperature of the neighbourhood of the insulator. The rain is sprayed at an angle of 45° on the insulator at the prescribed rate of 3 mm/minute. The resistivity of the water should be 100 ohm-m ± 10%. The prescribed voltage is maintained for one minute.

**Sample Tests** test the sample fully, up to and including the point of breakdown. These are done only on a few samples. An example for an insulator is given below.

**Electro-mechanical test**: The insulator is simultaneously subjected to electrical and mechanical stress. (i.e. it shall be subjected to a power frequency voltage and a tensile force simultaneously. The voltage shall be 75% of dry flash-over voltage of the unit. There should be no damage caused.

**Routine Tests** are to be applied routinely on all units to ensure a minimum level of performance. A typical test is given below.

**Power frequency withstand test**: In the case of insulators, testing shall be commenced at a low voltage and shall be increased rapidly until flash-over occurs every few seconds. The voltage shall be maintained at this value for a minimum of five minutes, or if failures occur, for five minutes after the last punctured piece has been removed. At the conclusion of the test the voltage shall be reduced to about one-third of the test voltage before switching off.