1. (a) for a transmission line \( \frac{\partial v}{\partial x} = l \frac{\partial i}{\partial t} , \quad \frac{\partial i}{\partial x} = c \frac{\partial v}{\partial t} \), and \( \alpha^2 \frac{\partial^2 v}{\partial x^2} = \frac{\partial^2 v}{\partial t^2} \) where \( \alpha^2 = 1/l.c \)

The solution to the partial differential equation is of the form

\[ v = f(x-at) + F(x+at) \]  

(show brief derivations)

Consider the function \( f(x-at) \)

at the point \( (x_0, t_0) \), function has value \( f (x_0 - \alpha t_0) \)

if it moves forward at constant velocity \( \alpha \),

then after time \( t \), it would have travelled a distance \( \alpha t \) so that its co-ordinates would be \( (x_0 + \alpha t, t_0 + t) \)

at the point \( (x_0 + \alpha t, t_0 + t) \), function has value \( f (x_0 + \alpha t - \alpha (t_0 + t)) \) or \( f (x_0 - \alpha t_0) \).

Thus the function \( f(x-at) \) remains constant when travelling forward at velocity \( \alpha \) and represents a forward travelling wave.

Similarly, \( F(x+at) \) represents a reverse travelling wave.

Thus the surge on a transmission line can be represented by a forward travelling wave and a reverse travelling wave \([3 \text{ marks}]\)

(b) The Bergeron's method is applied on a voltage-current diagram.
\( v = f(x-at) + F(x+at) \)

The corresponding surge current is given by

\[ i = \frac{1}{Z_0} \left[ f(x-at) - F(x+at) \right] \quad \text{or} \quad Z_0 \cdot i = [f(x-at) - F(x+at)] \]

Thus \( v + Z_0 \cdot i = f(x-at) = \text{constant for a forward wave} \)

Similarly, \( v - Z_0 \cdot i = F(x+at) = \text{constant for a reverse wave} \)

Thus forward waves and reverse waves can be represented by lines of slope \(-Z_0\) and \(+Z_0\) respectively on a \(v\) versus \(i\) diagram (Bergeron diagram shown above).

The initial point \( A_0 \) is obtained from the source characteristic and the surge impedance \( Z_0 \) corresponding to the first surge.

Thereafter successive reflections at \( B \) and \( A \) are considered by lines with slope \(-Z_0\) and \(+Z_0\) respectively. The voltage waveform can be projected and obtained as shown. [3 marks]

(c) for surge arriving along \( AJ \),

\[ \text{JB, JC, and JD effectively appear to be in parallel} \]

Thus effective combined \( Z_0 \)

\[ = \frac{400}{50/350} = 39.44 \Omega \]

\[ \therefore \quad \alpha = \frac{2Z_2}{Z_1 + Z_2} = \frac{2 \times 39.44}{400 + 39.44} = 0.1795 \]

and

\[ \beta = \frac{Z_2 - Z_1}{Z_1 + Z_2} = \frac{40.82 - 400}{400 + 40.82} = -0.8205 \]

\[ \therefore \quad \text{surge transmitted to JC} = 200 \times 0.1795 = 35.9 \text{ kV} \]

and \( \text{surge reflected to JA} = 200 \times (-0.8205) = -164.1 \text{ kV} \) [4 marks]
(d)

Travel time of AB = 120/300 = 400 μs, Travel time of BC = 45/300 = 150 μs

\[
\beta_{AB} = \frac{550 - 450}{550 + 450} = 0.1, \quad \beta_{CB} = -\beta_{AB} = -0.1, \quad \beta_{BC} = \frac{1450 - 550}{1450 + 550} = 0.45
\]

[1 mark]

Selecting the time step as 50 μs and setting AB proportional to 8 and BC to 3 gives

[4 marks]

[1 mark]
(e) In a high voltage line, when a line is energized from a source, a high voltage surge could travel with magnitudes exceeding twice the voltage appearing at the far end. Such switching surges could be reduced by the use of suitable switching resistors.

The switch is initially closed with the switching resistor R in series with the line, so that only a fraction of the original surge voltage would travel as a surge on the line.

When the original surge has significantly died down, the resistor R is short-circuited completing the switching process. This would then send only the balance voltage on to the line causing again a reduced surge. [2 marks]

(f) Inductances could be represented by considering them as very short lines with a distributed capacitance of negligible value to earth. This assumption will make the lumped element stub line to have negligible transmission times. It is usual to select the transmission times corresponding to the minimum time increment $\Delta t$.

If the travel time of the line is selected corresponding to $\tau = \Delta t$

$$\tau = \sqrt{\frac{L}{C}} \quad \text{so that} \quad C = \frac{\tau^2}{L} = \left(\frac{\Delta t}{L}\right)^2$$

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{L}{\tau} = \frac{L}{\Delta t}$$

Thus a lumped inductance may be represented by a stub line of transit time $\Delta t$ and surge impedance $L/\Delta t$. [2 marks]
2 (a) Lightning is an electric discharge in the form of a spark or flash originating in a charged cloud where the negative charge centre is generally located in the lower part of the cloud where the temperature is about \(-5^\circ\text{C}\), and that the main positive charge centre is located several kilometres higher up, where the temperature is usually below \(-20^\circ\text{C}\). In the majority of storm clouds, there is also a localised positively charged region near the base of the cloud where the temperature is \(0^\circ\text{C}\).

Fields of about 1000 V/m exist near the centre of a single bipolar cloud in which charges of about 20 C are separated by distances of about 3 km. Under the influence of sufficiently strong fields, large water drops become elongated in the direction of the field and become unstable, and streamers develop at their ends with the onset of corona discharges. Drops of radius 2 mm develop streamers in fields exceeding a 9 kV/cm - much less than the 30 kV/cm required to initiate the breakdown of dry air. The high field need only be very localised, because a streamer starting from one drop may propagate itself from drop to drop under a much weaker field.

When the electric field in the vicinity of one of the negative charge centres builds up to the critical value (about 10 kV/cm), an ionised channel (or streamer) is formed, which propagates from the cloud to earth with a velocity that might be as high as one-tenth the speed of light. Usually this streamer is extinguished when only a short distance from the cloud.
Forty micro-seconds or so after the first streamer, a second streamer occurs, closely following the path of the first, and propagating the ionised channel a little further before it is also spent. This process continues a number of times, each step increasing the channel length by 10 to 200 m. Because of the step like sequence in which this streamer travels to earth, this process is termed the **stepped leader** stroke.

When eventually the stepped leader has approached to within 15 to 50 m of the earth, the field intensity at earth is sufficient for an upward streamer to develop and bridge the remaining gap. A large **neutralising** current flows along the ionised path, produced by the stepped leader, to neutralise the charge. This current flow is termed the **return** stroke and may carry currents as high as 200 kA, although the average current is about 20 kA.

Following the first, or main stroke and after about 40 ms, a second leader stroke propagates to earth in a continuous and rapid manner and again a return stroke follows. This second and subsequent leader strokes which travel along the already energised channel are termed **dart leaders**.

What appears as a single flash of lightning usually consist of a number of successive strokes, following the same track in space, at intervals of a few hundredths of a second. The average number of strokes in a multiple strike is four, but as many as 40 have been reported. The time interval between strokes ranges from 20 to 700 ms, but is most frequently 40-50 ms. The average duration of a complete flash being about 250 ms.

(b) Figure shows the earth wire coverage of a transmission line against direct strikes.

The region (1) represents the region in which lightning will most likely strike the earth wire and thus provide protection against direct strikes. The locus of the lower boundary of this region is approximately defined by the perpendicular bisector of the line joining the phase conductor (the outermost for a horizontal arrangement and the uppermost in the case of a vertical arrangement) and the earth wire.

The region (2) represents the region in which lightning avoids both the overhead conductor as well as the earth wire but strikes some nearby object. The region has the upper boundary defined approximately by a parabolic locus. This locus is taken as equidistant from both the earth plane as well as the phase conductor. (This assumption is not exactly true as the phase conductor is a better attractor of lightning due to its sharper configuration).

Depending on the strength of the charge on the leader core, lightning will be initiated at a distance away from the object struck. Thus if the leader core could approach very close to the phase conductor before it discharges, then that particular stroke will be weak. This defines a minimum region within which lightning strikes terminating on the line does not do any damage. This region thus has a circular locus around the conductor, which is not be considered in risk evaluation.

The region (3) is the balance region, demarcated by the locus of region (1), the locus of region (2) and the circular locus where the stroke is too weak to cause damage. In this region (3) the lightning stroke is most likely to terminate on the phase conductor. The area (3) is thus a measure of the efficiency of the earth-wire protection. The smaller this region is the better the shielding provided by the overhead earth wires. For the same semi-vertical shielding angle $\theta$, the taller the tower the lesser the efficiency of protection provided by the earth wire. Further if the semi-vertical angle of shielding is reduced, the area (3) reduces giving better protection. Thus to obtain the same degree of protection, taller towers require smaller protection angles.
(c) The area of attraction of the transmission line is based on a gradient of 3 kV/cm at the tower at which the upward streamer is initiated from the tower. It has been found that for the average stroke the protective ratio is approximately two for a tower.

\[ R \sim 2H \text{ at } 20 \text{kA} \]

The area of attraction of a lightning conductor may be expected to be equal to an area around the base of the conductor with a radius of twice the conductor height. In the case of transmission lines, the earth wire is positioned to protect the phase conductors against lightning strokes and hence it is a protective conductor. However, the earth wire attracts strokes that would not normally terminate on the line. Similarly, phase conductors themselves attract lightning strokes and it is hardly correct to talk of the protective zone. A more appropriate term is the area of attraction. Figure shows the area of attraction of the transmission line and towers.

Since the tower is like a lightning conductor, the area of attraction of the tower can be taken as equal to a circle with radius twice the tower height. An earth wire is more uniform than a transmission tower, in that it does not have a sharp point but a sharp line. It has been estimated that an area either side of the earth wire to a distance of 1.5 or 1.6 times the effective height of the earth wire multiplied by the length of the earth wire is a reasonable value to be taken. Further it must be noted that due to the sag of the earth wire, the effective height of the earth wire is itself only about 80% of the height at the tower. Thus a distance of 64% of the radius of attraction at the tower may be taken for the attraction distance of the earth wire. The phase conductor may be treated similarly, but with the height of the phase conductor being considered instead of that of the earth wire. [3 marks]
For a B.I.L of 550 kV, and an insulation margin of 20%,
Maximum permissible voltage = 550 x 80/100 = 440 kV.
Since the voltage is increased by the transmission coefficient 1.667 at the terminal equipment, the
maximum permissible incident voltage must be decreased by this factor.
maximum permissible incident surge = 440/ 1.667 = 264 kV
distortion caused must reduce the surge to a magnitude of 264 kV.
∴ 1000 e^{-0.05t} = 264.
delay time \( t_1 = \Delta t = \frac{-\ln (0.264)}{0.05} = 26.64 \mu s. \)
thus \( \frac{26.64}{x} = \frac{1}{110} \times (1 - \frac{200}{264}) \)
x = 12088 m = 12 km.
Thus the minimum length of shielding wire required is 12 km. [6 marks]

(b) For 132 kV, maximum value of system rms voltage = 145 kV
\[ \therefore \text{voltage rating for effectively earthed system} = 145 \times 0.8 = 116 \text{ kV} \]
Line insulation limits the surges transmitted to 950 kV.
Since the line \( Z_0 = 400 \Omega, \) and transformer \( Z_0 = 1600 \Omega, \)
transmission coefficient = \( \frac{2 \times 1600}{400 + 1600} = 1.6 \)
Assuming and an arrestor residual discharge voltage of \( E_a, \) the surge current and hence the arrestor
discharge current would be given by
\[ I_a = \frac{1.6 E - E_a}{Z_0} = \frac{1.6 \times 950 - E_a}{400} \]
\( E_a \) range from 316 kV to 418 kV.
For this \( I_a \) has the range 2.76 to 3.01 kA.
Thus the required discharge current for the arrestor is 5 kA. [8 marks]
(c) In a statistical study, what has to be known is not the highest overvoltage possible, but the statistical distribution of overvoltages. The switching overvoltage probability is shown. It is seen that probability of overvoltage decreases very rapidly.

At the higher transmission voltages, the clearances in air do not increase linearly with voltage but approximately to $V^{1.6}$. Thus, while it may be economically feasible to protect the lower voltage lines up to a high overvoltage factor of 3.5 (say), it is not economically feasible to have such high overvoltage factors on the higher voltage lines. In the higher voltage systems, it is the switching overvoltages that is predominant and these may be controlled by proper design of switching devices or by the use of surge diverters set to operate on the higher overvoltages. In such cases, the failure probability would be extremely low.

The aim of statistical methods is to quantify the risk of failure of insulation through numerical analysis of the statistical nature of the overvoltage magnitudes and of electrical withstand strength of insulation.

The risk of failure of the insulation is dependant on the integral of the product of the overvoltage density function $f_0(V)$ and the probability of insulation failure $P(V)$. Thus the risk of flashover per switching operation is equal to the area under the curve $\int f_0(V)P(V)\,dV$.

<table>
<thead>
<tr>
<th>Probability %</th>
<th>99</th>
<th>95</th>
<th>90</th>
<th>80</th>
<th>50</th>
<th>20</th>
<th>10</th>
<th>5</th>
<th>1</th>
<th>0.1</th>
<th>0.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overvoltage (pu)</td>
<td>1.0</td>
<td>1.5</td>
<td>2.0</td>
<td>2.5</td>
<td>3.0</td>
<td>3.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Since we cannot find suitable insulation such that the withstand distribution does not overlap with the overvoltage distribution, in the statistical method of analysis, the insulation is selected such that the 2% overvoltage probability coincides with the 90% withstand probability as shown. [4 marks]

(d) Description of one form of surge diverter with suitable sketches [2 marks]
4 (a) assuming a voltage efficiency of 95% for the impulse generator,
Minimum number of stages required = \( \frac{200/(25\sqrt{2})}{0.95} = 5.95 \)
Select 6 stages [1 mark]

(b) Nominal Voltage = 200 kV, Nominal energy = 1 kJ

The impulse generator can be reduced to the form

\[ \therefore \frac{1}{2} C_1 \times 200^2 = 1000 \rightarrow C_1 = 0.05 \mu F = 50 \text{nF} \] [1 mark]

During wavefront, since \( R_1 >> R_2 \),
approximate charging circuit is giving a charging time constant

\[ \frac{1}{\beta} = R_2 \times \left( C_1/C_2 \right) = \frac{R_2 C_1 C_2}{(C_1 + C_2)} = \eta R_2 C_2 \]

where, voltage efficiency = \( \eta = \frac{C_1}{C_1 + C_2} \), and \( v = V_{\text{max}} \left( 1 - e^{-\beta t} \right) \)

\[ \text{i.e. } 0.95 = \frac{C_1}{C_1 + C_2} , \rightarrow C_2 = 0.00263 \mu F = 2.63 \text{nF} \] [2 marks]

defining wavefront based on 30% to 90% and extrapolation for the standard waveform, \( 1.2/50 \mu s \)

\[ t_f = \frac{1}{0.90 - 0.30} \times (t_{90} - t_{30}) = \frac{1}{0.60} \times (t_{90} - t_{30}) = 1.2 \mu s \rightarrow (t_{90} - t_{30}) = 0.72 \mu s \]

\[ 0.3 \ V_m = V_m \left( 1 - e^{\beta t_{30}} \right) \text{ giving } 0.7 = e^{\beta t_{30}} \]

\[ 0.9 \ V_m = V_m \left( 1 - e^{\beta t_{90}} \right) \text{ giving } 0.1 = e^{\beta t_{90}} \]

therefore, \( 7 = e^{\beta (t_{90} - t_{30})} \) giving \( t_{90} - t_{30} = (\ln 7)/\beta = 0.72 \)

\[ \beta = (\ln 7)/0.72 \ (\mu s)^{-1} = 2.70 \ (\mu s)^{-1} \]

\[ \therefore \ 0.95 \ R_2 \ C_2 = 1/2.7 \rightarrow R_2 = \frac{1}{2.70 \times 0.95 \times 0.00263} = 148.2 \ \Omega \] [2 marks]
Similarly, during wavetail, since $R_2 \ll R_1$, the approximate charging circuit is giving a discharging time constant

$$1/\alpha = R_1 .(C_1+C_2) = R_1 C_2 / \eta$$

and an expression $v = V_{\text{max}} e^{-\alpha t}$

at wavetail $0.5 V_m = V_m e^{-\alpha t}$ giving $\alpha t_t = \ln(2)$, $t_t = 50 \mu s$

therefore $\alpha = 0.693/ t_t = 0.693/50 = 0.01386 \, (\mu s)^{-1}$

i.e. $R_1 = 1/((0.05+0.00263)*0.01386) = 1371 \, \Omega = 1.37 \, k\Omega$ [2 marks]

or 6 resistors or $1.37/6 = 228 \, \Omega$

Thus the components of the circuit are

1 wavefront control resistor = $148.2 \, \Omega$

6 wavetail control resistors each of value = $228 \, \Omega$

6 capacitors each of value $(6C_1) = 300 \, nF$

1 capacitor of value $(C_2) = 2.63 \, nF$

Select the charging resistors at least 1000 larger than the wavetail control resistors

charging resistors each of value = $1 \, M\Omega$ [1 mark]

(c)
[Alternate solution methods]

Using standard derived formulae,

Energy \( W = 2 \, C_1 V_1^2 \)

Nominal Voltage = 200 kV, Nominal energy = 1 kJ,

\[ \therefore \frac{1}{2} C_1 \cdot 200^2 = 1000 \rightarrow C_1 = 0.05 \ \mu F = 50 \ nF \]

\[ \eta = \frac{C_1}{C_1 + C_2} = 0.95, \rightarrow C_2 = 0.00263 \ \mu F = 2.63 \ nF \quad (95\% \text{ efficiency assumed}) \]

with rise time based on 10\% to 90\% of peak for wavefront

\( t_f = 2.75 \, \eta \, R_2 \, C_2 \)

\( 1.2 = 2.75 \cdot 0.95 \cdot R_2 \cdot 0.00263, \rightarrow R_2 = 174.65 \ \Omega \)

OR using derived formulae, with rise time based on 30\% to 90\% of peak for wavefront

\( t_f = 3.243 \, \eta \, R_2 \, C_2 \)

\( 1.2 = 3.243 \cdot 0.95 \cdot R_2 \cdot 0.00263, \rightarrow R_2 = 148.1 \ \Omega \)

For wavetail

\( t_t = \frac{0.693 \, R_1 \, C_1}{\eta} \)

\( 50 = 0.693 \cdot R_1 \cdot 50/0.95, \rightarrow R_1 = 1.371 \ k\Omega \)

The remaining analysis is the same as before and is not repeated here.
It is seen from the waveform that the peak value is 100 kV → magnitude [1 mark]
and that the value decreases to the half value of 50 kV in 50 μs → wavetail time [1 mark]

Considering 30% value and 90% value and extrapolating,
wavefront time = 1.1 – (– 0.1) = 1.2 μs [3 marks]

[alternate considering 10% to 90% and extrapolating gives wavefront time = 1.15 μs]
(e) The likely reason for the oscillations in the wavefront is that the cable connecting the oscilloscope to the impulse generator has not been matched. [1 mark]

The period of the oscillations in the wavefront appears to be $0.45/5 = 0.09 \mu s$.

Period corresponds to twice the time of travel of the cable. ∴ travel time $= 0.025 \mu s$.

Assuming velocity of propagation in cable as $2 \times 10^5 \text{ m/s}$ or $200 \text{ m/\mu s}$.

Likely length of cable $= 200 \times 0.025 = 5 \text{ m}$ [1 mark]

(f) Matching of the cable is undertaken both at the sending end and the receiving end to avoid any possible reflections. [1 mark]