University of Moratuwa, Sri Lanka
Faculty of Engineering
Department of Electrical Engineering
B. Sc. Engineering Honours Degree Course
Level 2 – Semester 1 Examination

EE201 – THEORY OF ELECTRICITY

Time Allowed:  3 Hours  August 2006

Additional Material
Graph Paper will be provided if required.

Instructions to Candidates
This paper contains 7 questions in 4 pages.
This examination accounts for 70% of the module assessment.
Total marks for the paper is 70 marks.
The maximum mark attainable for each part is indicated in square brackets.
Answer All Questions.
This is a closed book examination and only authorised Calculators will be permitted.

Technical Information for candidates
Permeability of free space $\mu_0 = 4 \pi \times 10^{-7}$ H/m
Permittivity of free space $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m
Velocity of light in free space $= 2.998 \times 10^8$ m/s
Question 1

(a) A certain alternating voltage source has an internal emf of 240 V at 50 Hz, an internal impedance of \((10+j10)\ \Omega\) and supplies a load of \((R + jX)\). If the terminal voltage is to kept at 230 V, write down an equation to relate the magnitudes of the internal emf and the terminal voltage in terms of \(R\) and \(X\). \[2\] marks

(b) Determine the value of the \(R\) and \(X\) required to transfer maximum power to the load. \[5\] marks

(c) Determine the values of this maximum power, and the load current under these conditions. \[1\] mark

(d) Draw the phasor diagram showing the internal emf, the load voltage, voltages across \(R\) and \(X\) and the current under these conditions. \[2\] marks

Question 2

(a) Briefly explain three significantly different methods of defining the resonance frequency of an R-L-C circuit. \[2\] marks

(b) Figure Q2bc shows a circuit which is mutually coupled. Using first principles, convert it to a non-coupled circuit. \[3\] marks

(c) Using the result of Q2(b) or otherwise, determine the resonance frequency and the current at resonance. \[2\] marks

(d) Determine the frequencies at the half-power points. \[3\] marks

Question 3

(a) For the circuit shown in figure Q3-4, find the Thevenin’s equivalent circuit at the broken line \(B\). \[4\] marks

(b) Determine the z-parameter two-port matrix for the network shown in figure Q3-4 with the port 1 taken at the broken line at \(A\) and the port 2 taken at the broken line at \(B\). \[4\] marks
Question 4

(a) For the circuit shown in figure Q3-4, write down the branch-mesh incidence matrix and the branch impedance matrix. [2 marks]

(b) Show how the mesh impedance matrix may be obtained from the branch-mesh incidence matrix and the branch impedance matrix. Hence or otherwise write down the mesh impedance matrix. [2 marks]

(c) Using matrix mesh analysis, determine the mesh currents. [3 marks]

(d) Determine the currents in all the branches in figure Q3-4. [3 marks]

Question 5

A 3 phase, 50 Hz, 3-wire, balanced supply feeds, with a voltage of 400 V at the load end, (i) a balanced star connected load with each arm consisting of a series combination of an inductor L = 100 mH and a resistor R = 50 Ω, and (ii) a balanced three phase motor of rated at 1 kW running at full load at power factor 0.6 lag.

Determine

(a) the total line current supplied from the source [3 marks]

(b) the total reactive power supplied. [1 mark]

If each of the lines supplying the load has an impedance (l = 10 mH, r = 5 Ω), determine

(c) the voltage at the supply end of the line, [2 marks]

(d) the overall power factor at the source, and [1 mark]

(e) the capacitances that must be connected across the combined load to improve the power factor to 0.9 lag when the load voltage is 400 V. [3 marks]

Question 6

(a) Write down the matrix equation relating the phase voltages with the corresponding sequence components. [1 mark]

(b) Determine the phase components corresponding to the sequence components. [3 marks]

Positive sequence voltage = 200∠0° V
Negative sequence voltage = 100∠−30° V
Zero sequence voltage = 50∠90° V

(c) Using the results obtained in 6(b), show graphically, how the positive sequence component may be determined. [2 marks]
Question 7

(a) Very briefly explain the link between the Fourier Series, Fourier Transform and the Laplace Transform. [2 marks]

(b) Determine the Fourier Series of the periodic waveform \( i(t) \) shown in Figure Q7b to 3 significant terms. [6 marks]

\[
\begin{align*}
  i(t) &= 100 \cos 314t & \text{for } 0 < t \leq 0.005 \\
  i(t) &= 100 - 20000t & \text{for } 0.005 < t \leq 0.01 \\
  i(t) &= 300 - 20000t/ & \text{for } 0.01 < t \leq 0.015 \\
  i(t) &= -100 \cos 314t & \text{for } 0.015 < t \leq 0.02
\end{align*}
\]

(c) If a current \( i(t) = 10 \sin 314t + 75 \sin 628t - 15 \sin 942t \) is supplied to a series RL circuit (\( R = 10 \, \Omega \), \( L = 10 \, \text{mH} \)), determine an expression for the corresponding voltage across the circuit. [1 mark]

(d) Determine the Laplace Transform of the following:
   a. causal ramp waveform \( t \) [1 mark]
   b. causal step waveform \( h(t) \) [1 mark]
   c. causal sine waveform \( \sin \omega t \) [1 mark]
   d. causal waveform \( v(t) \) shown in Figure Q7d. [3 marks]

\[
\begin{align*}
  v(t) &= 10000t & \text{for } 0 < t \leq 0.01, \\
  v(t) &= 100 \sin 50\pi t & \text{for } 0.01 < t \leq 0.03
\end{align*}
\]

(e) If the voltage waveform \( v(t) \) shown in Figure Q7d is applied across a series RC circuit (\( R = 10 \, \Omega \), \( C = 10 \, \mu\text{F} \)), show how an expression for the Laplace transform of the corresponding current in the circuit is determined. [1 mark]