University of Moratuwa, Sri Lanka
Faculty of Engineering
Department of Electrical Engineering
B. Sc. Engineering Honours Degree Course
Level 2 – Semester … Examination

EE2010 – THEORY OF ELECTRICITY

Time Allowed: 3 Hours ……… 2009

Additional Material
Graph Paper will be provided if required.
A table of Laplace transforms is provided on the other side of this page.

Instructions to Candidates
This paper contains 7 questions in 6 pages, including the cover page.
Answer All Questions.
This examination accounts for 70% of the module assessment.
Each question carries a total 10 marks. Maximum marks allocated for each part of a question is indicated in square brackets at the end of the part.
Total allocation for the paper is 70 marks.
This is a closed book examination and only authorised calculators will be permitted.

Technical Data:
Permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ H/m
Permittivity of free space $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m
Velocity of light in free space $= 2.998 \times 10^8$ m/s
Table of Laplace Transforms of common causal functions $f(t)$

<table>
<thead>
<tr>
<th>$f(t)$</th>
<th>$F(s) = L[f(t)]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit impulse – $\delta t$</td>
<td>1</td>
</tr>
<tr>
<td>Unit step – $U(t)$</td>
<td>$\frac{1}{s}$</td>
</tr>
<tr>
<td>$t$</td>
<td>$\frac{1}{s^2}$</td>
</tr>
<tr>
<td>$t^n$</td>
<td>$\frac{n!}{s^{n+1}}$</td>
</tr>
<tr>
<td>$e^{-at}$</td>
<td>$\frac{1}{(s+a)}$</td>
</tr>
<tr>
<td>$1 - e^{-at}$</td>
<td>$\frac{a}{s(s+a)}$</td>
</tr>
<tr>
<td>$te^{-at}$</td>
<td>$\frac{1}{(s+a)^2}$</td>
</tr>
<tr>
<td>$t^n e^{-at}$</td>
<td>$\frac{n!}{(s+a)^{n+1}}$</td>
</tr>
<tr>
<td>$e^{-at} - e^{-bt}$</td>
<td>$\frac{b-a}{(s+a)(s+b)}$</td>
</tr>
<tr>
<td>$\sin(\omega t)$</td>
<td>$\frac{\omega}{s^2 + \omega^2}$</td>
</tr>
<tr>
<td>$\sin(\omega t + \phi)$</td>
<td>$\frac{\omega \cos(\phi) + s \sin(\phi)}{s^2 + \omega^2}$</td>
</tr>
<tr>
<td>$t \sin(\omega t)$</td>
<td>$\frac{2 \omega s}{s^2 + \omega^2}$</td>
</tr>
<tr>
<td>$\cos(\omega t)$</td>
<td>$\frac{s}{s^2 + \omega^2}$</td>
</tr>
<tr>
<td>$\cos(\omega t + \phi)$</td>
<td>$\frac{s \cos(\phi) - \omega \sin(\phi)}{s^2 + \omega^2}$</td>
</tr>
<tr>
<td>$t \cos(\omega t)$</td>
<td>$\frac{s^2 - \omega^2}{(s^2 + \omega^2)}$</td>
</tr>
<tr>
<td>$e^{-at} \sin(\omega t)$</td>
<td>$\frac{\omega}{(s+a)^2 + \omega^2}$</td>
</tr>
<tr>
<td>$e^{-at} \cos(\omega t)$</td>
<td>$\frac{s + a}{(s+a)^2 + \omega^2}$</td>
</tr>
<tr>
<td>$\sinh(\omega t)$</td>
<td>$\frac{\omega}{s^2 - \omega^2}$</td>
</tr>
<tr>
<td>$\cosh(\omega t)$</td>
<td>$\frac{s}{s^2 - \omega^2}$</td>
</tr>
</tbody>
</table>
**Question 1**

(a) Figure Q1a shows a practical operational amplifier with a resistor R and a capacitor C connected as shown.

i. Draw the complete equivalent circuit showing also the input resistance $R_{in}$, output resistance $R_{out}$, and open loop gain $A$. [1 mark]

ii. Obtain expressions relating the output voltage $V_{out}$ with the input voltage $V_{in}$. [2 marks]

iii. If the operational amplifier is ideal, derive a simplified relationship between $V_{out}$ and $V_{in}$. [2 marks]

(b) A circuit consists of a series combination of a 100 mH inductance with a winding resistance of 10 $\Omega$, and a 100 $\mu$F capacitor with an effective parallel resistance of 1000 $\Omega$. Calculate the current supplied and the voltage across the capacitor when supplied at 200V an angular frequency 250 rad/s. Sketch the phasor diagram showing these voltages and the current. [3 marks]

(c) If the supply frequency is varied from 250 rad/s, at determine the frequency at which resonance would occur, and the circuit current at resonance. [2 marks]

**Question 2**

(a) Figure Q2a shows the magnetic circuit of a three phase transformer. If $I_A = 20 \angle 0^\circ$ A and the currents in the three phases are balanced, draw the magnetic equivalent circuit, with values, for the calculation of flux, if N=100 turns in each coil, the effective cross-section of each limb is $A = 2$ cm$^2$, the effective magnetic lengths of the outer limb $l_o=20$ cm and of the middle limb $l_m=6$ cm. The relative permeability of the magnetic material is $\mu_r = 2000$, and the transformer is operating in the linear region. [4 marks]

(b) Figure Q2b shows an a.c. circuit with ideal components.

i. If Z is inductance with a reactance of 40 $\Omega$, using Thevenin’s theorem, determine the current in the inductor. [3 marks]

ii. What would be the corresponding current if Z is a 40 $\Omega$ resistor? [1 mark]

iii. What is the value of Z that will extract the maximum active power, and what is the value of this maximum power? [2 marks]
**Question 3**

(a) For the 3-phase network shown in figure Q3a, the current $I_{L3}$ is measured to be $3.884\angle 87.02^0$ A. Using compensation theorem, determine the new value of the current, if $R_3$ is decreased to 48 Ω. [2 marks]

(b) For the circuit shown in figure Q3b, determine from first principles, a non-coupled equivalent circuit. [2 marks]

(c) For the network shown in figure Q3cd, determine the two-port admittance matrix [3 marks]

(d) For the network in Fig Q3c determine also the characteristic impedance $Z_o$, which when connected at port 2 as a load would give the input impedance at port 1 also as $Z_o$. [3 marks]

**Question 4**

For the circuit shown in figure Q4,

(a) Draw the oriented graph of the network. Using a suitable tree, draw the corresponding loops. [2 mark]

(b) Write down the

i. branch impedance matrix [1 mark]

ii. branch source voltage vector [1 mark]

iii. branch-mesh incidence matrix [1 mark]

(c) Hence obtain the mesh-impedance matrix and the mesh voltage vector. [2 mark]

(d) If $E_1 = E = E_2$ obtain the currents in all the branches. [3 mark]
**Question 5**

(a) A balanced 3-phase, 400 V, 50 Hz supply feeds a 3-phase star-connected balanced load consisting of arms of value \((80 + j60) \Omega\) each. Determine the current drawn, the active power consumed and the power factor of the load. [2 marks]

(b) A 3-phase motor consuming 1 kW at a power factor of 0.72 lag is added to the system. Determine the line current supplied from the source, and the supply power factor. [1 mark]

(c) It is now required to improve the load power factor to 0.95 lag by connecting a bank of delta-connected capacitors. Determine the rating of each capacitor. [2 marks]

The above balanced supply, with phase sequence ABC and phase A voltage taken as reference, now feeds an unbalanced load of \(I_A = 10.0 \angle -30^\circ\)A, \(I_B = 4.0 \angle -120^\circ\)A and \(I_C = 6.0 \angle 115^\circ\)A.

(d) Determine the symmetrical components of the currents in phase A. [3 marks]

(e) Determine also the power associated with each sequence component [2 marks]

**Question 6**

![Figure Q6a](image)

(a) Figure Q6a shows a periodic waveform observed on a digital oscilloscope screen, with 1 div on the horizontal scale corresponding to 2.5 ms.

i. Determine the period of the waveform. [1 mark]

ii. Identify any symmetrical properties in the waveform, and how you may make use of it in the Fourier analysis of the waveform. [1 mark]

iii. Identify the most dominant harmonic frequency in the waveform. [2 marks]

(b) A voltage \(v(t) = 100 \sin 250t\) volt when applied across a certain non linear circuit produces a current \(i(t) = 20 + 10 \sin (250t + \pi/3) - 4 \sin (750t - \pi/6)\) ampere.

i. Determine the rms value of the current \(i(t)\) [2 marks]

ii. Determine the average power supplied from the source. [2 marks]

(c) If the current \(i(t)\) passes through a series combination of \(L = 10\) mH and a resistance of 1 \(\Omega\), determine the rms value of the voltage drop across the combination [2 marks]
Question 7

(a) Determine from first principles the Laplace transform of (i) the unit step $H(t)$, where $H(t) = 0$ for $t<0$ and $H(t) = 1$ for $t \geq 0$, and (ii) a unit ramp $R(t)$, where $R(t) = 0$ for $t<0$ and $R(t) = t$ for $t \geq 0$.\[2 \text{ marks}\]

(b) A function $f_i(t)$ is defined for the period $0 < t \leq T$, and its Laplace transform is $F_i(s)$. Determine from first principles, the Laplace transform $F(s)$ of the repetitive waveform $f(t)$ with period $T$, where $f(t) = f_i(t)$ for $0 < t \leq T$\[2 \text{ marks}\]

(c) Using the result of Q7(a), determine the Laplace transform of the causal repetitive sawtooth waveform $s(t)$ shown in figure Q7c.\[4 \text{ marks}\]

(d) Sketch the Laplace transformed equivalent circuit for the circuit shown in figure Q7d, if the switch shown is closed at time $t=0$.\[2 \text{ marks}\]